Code No. 1054 / CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Mathematics

Paper – I: Complex Analysis

Time: 2 Hours **Max.Marks: 80**

PART – A (4x5 = 20 Marks)

Answer any four questions.

1 Show that an analytic function cannot have a constant absolute value without reducing to a constant.

2 Define cross ratio. Prove that cross ratio is invariant under a linear transformation.

3 Compute $\int_{|z|=1} \frac{e^{2}}{z}\, dz$.

4 If γ is the piecewise differentiable closed curve that does not pass through the point, a, then, prove that $\int_{\gamma} \frac{dz}{z-a}$ is the integral multiple of $2\pi i$.

5 Prove that an analytic function comes arbitrarily close to any complex value in every neighbourhood of an essential singularity.

6 If z=a is a pole of order m for f(z), then prove that

$$\operatorname*{Res}_{z=a} f(z) = \frac{1}{(m-1)!} \operatorname*{lt}_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left[(z-a)^m f(z) \right]$$

7 Prove that $\frac{\pi^2}{\sin^2 \pi z} = \sum_{n=-\infty}^{\infty} \frac{1}{(z-n)^2}$.

8 A necessary and sufficient condition for the absolute convergence of the product $\prod_{n=1}^{\infty} (1 + a_n)$ is the convergence of the series $\sum_{n=1}^{\infty} |a_n|$.

PART – B (4x15 = 60 Marks)
Answer any four questions.

9 State and prove the necessary condition for analytic function.

10 Prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.

11 State and prove Cauchy's theorem for a rectangle.

12 i) State and prove Cauchy's integral formula.

ii) Evaluate $\int_{|z|=1} \frac{e^z}{z} dz$.

13 Evaluate $\int_0^{\pi/2} \frac{1}{1+\sin^2\theta} d\theta$

14 Evaluate $\int_0^{\infty} \frac{1}{1+x^4} dx$

15 State and prove argument principle.

16 State and prove the mean value property of harmonic functions.

17 State and prove Mittog-Leffler theorem.

18 State and prove Legendre's duplication formula.

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Code No.1059 / CDE / BL

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Mathematics

Paper – I: Topology & Functional Analysis

Time: 2 Hours **Max.Marks: 100**

Answer any four questions. **(4x25 = 100 Marks)**

1 a) Prove that every second countable space is separable.
b) Prove that every separable metric space is second countable.

2 a) Prove that any closed subspace of a compact space is compact.
b) State and prove Lebesgue covering lemma.

3 State and prove Ascoli's theorem

4 a) Let X be a T_1 – space. Then prove that X is normal if and only if each neighbourhood of a closed set F contains the closure of some neighbourhood of F.
b) State and prove Urysohn's lemma.

5 a) Prove that a topological space X is disconnected if and only if there exists a continuous mapping of X onto the discrete two point space {0,1}.
b) Define component of a space. If $\{A_i\}$ is a non-empty class of connected subspaces of a topological space X such that $\cap_i A_i \neq \phi$, then prove that $A = \cup_i A_i$ is connected.

6 a) Let N be a normed linear space and x_0 be a non-zero vector of N. Then prove that there exists a bounded linear functional f_0 on N such that
$f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$.
b) State and prove closed graph theorem.

7 a) State and prove uniform boundedness theorem.
b) Prove that a non-empty subset X of a normed linear space N is bounded if and only if $f(x)$ is a bounded set of number for each $f \in N^*$.

8 a) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.
b) Prove that if M is a proper closed linear subspace of a Hilbert space H, then there exists a non-zero vector z_0 in H such that $z_0 \perp$ M.

9 Let H be a Hilbert space and let $\{e_i\}$ be an orthonormal set in H. Then prove that the following conditions are all equivalent to one another:

i) $\{e_i\}$ is complete

ii) $x \perp \{e_i\} \Rightarrow x = 0$

iii) If x is an arbitrary vector in H, then $x = \sum (x, e_i) e_i$.

iv) If x is an arbitrary vector in H, then $\|x\|^2 = \sum |(x, e_i)|^2$.

10 a) Prove that if T is an operator on Hilbert space H for which $(Tx, x) = 0$ for all x, then T=0.

b) If $p_1, p_2, ..., p_n$ are the projections on closed linear subspaces $M_1, M_2, \ldots, M_n$ of Hilbert space H, then prove that $P = P_1 + P_2 + \ldots P_n$ is a projection $\Leftrightarrow$ the $P_i^{'}s$ are pairwise orthogonal and in this case P is he projection on $M = M_1 + M_2 + \ldots M_n$.

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Code No. 1069 / CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Statistics

Paper – I: Statistical Inference

Time: 2 Hours **Max.Marks: 80**

PART – A (4x5 = 20 Marks)
Answer any four questions.

1 Define critical region and type-I, type-II errors.
2 Explain randomized and non-randomized tests.
3 What are the advantages of sequential analysis?
4 Define general form of OC function and give its importance.
5 Explain the concept of robustness in inference.
6 Explain one sample run test.
7 Explain ARE of one sample location test.
8 Define loss function and risk function and give their importance.

PART – B (4x15 = 60 Marks)
Answer any four questions.

9 Discuss consistency and unbiasedness of tests.

10 Distinguish between interval estimation and testing of hypothesis.

11 Show that SPRT terminates with probability one.

12 Explain SPRT procedure for binomial distribution and obtain OC, ASN functions.

13 Discuss Wilcoxon – Mann Whitney test for two sample problem.

14 Explain Chi-square test for goodness of fit and independence of attributes.

15 Explain estimation as a statistical decision problem.

16 Discuss about admissible decision function and optimal decision function.

17 Explain Ansari-Bradley two sample test procedure.

18 Derive OC and ASN functions in SPRT.

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Code No. 1055/CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Mathematics

Paper – II: Measure Theory

Time: 2 Hours **Max.Marks: 80**

PART – A (4x5 = 20 Marks)
Answer any four questions.

1 Define a σ – algebra of sets in X and furnish an example.

2 Prove that every countable set has outer measure zero.

3 Define absolute continuity of a function f defined on [a,b].

4 If f is of bounded variation on [a,b], then prove that $T_a^b = P_a^b + N_a^b$.

5 Define a measure space and give an example.

6 Define a positive set with respect to a signed measure and give an example.

7 Define a measurable rectangle.

8 Define an outer measure induced by a measure on an algebra.

PART – B (4x15 = 60 Marks)
Answer any four questions.

9 State and prove monotone convergence theorem.

10 State and prove Fatou's lemma.

11 State and prove Holder's inequality.

12 Prove that a normed linear space X is complete $\Leftrightarrow$ every absolutely summable series is summable.

13 State and prove Hahn's decomposition theorem.

14 State and prove Jordan's decomposition theorem.

15 State and prove Tonelli's theorem.

16 State and prove Caratheodory extension theorem.

17 If f is integrable on [a,b], then prove that the function defined by

$F(x) = \int_a^x f(t)\,dt$ is a continuous function of bounded variation on [a,b].

18 Prove that the L^P-spaces are complete.

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Code No. 1055/O/CDE/BL

FACULTY OF SCIENCE

M. Sc. (Final) CDE Examination, November 2020

Subject : Mathematics

Paper – II : Measures Theory

Time : 2 Hours **Max. Marks: 100**

Note: Answer any four questions. **(4x25=100 Marks)**

1 (a) State and prove monotone convergence theorem.
(b) State and prove Fatou's lemma.

2 (a) Construct a set which is non-measurable in the sense of Lebesgue.
(b) State and prove bounded convergence theorem.

3 (a) State and prove Holder's inequality.
(b) Prove that a normed linear space X is complete $\Leftrightarrow$ every absolutely summable series is summable.

4 Show that L^p – spaces are complete.

5 (a) State and prove Hahn's decomposition theorem.
(b) State and prove Jordan's decomposition theorem.

6 State and prove generalized Fatou's lemma.

7 State and prove Caratheodory extension theorem.

8 State and prove Fubini's theorem.

9 State and prove Riesz-Fischer theorem.

10 (a) If f is of bounded variation on [a, b], than prove that $T_a^b = P_a^b + N_a^b$.
(b) Prove that the set of all μ^* - measurable sets in X is a σ- algebra.

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Code No. 1070 / CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Statistics

Paper – II: Linear Models and Designs of Experiments

Time: 2 Hours **Max.Marks: 80**

PART – A (4x5 = 20 Marks)
Answer any four questions.

1 What is multiple regression model? How do you estimate its parameters?

2 Define multiple and partial correlation coefficients. How do you compute in case X_1 is dependent and X_2, X_3 independent variables?

3 Define Latin square design and give its layout for 6 treatments.

4 State Cochran's theorem on quadratic forms. Discuss its use in analysis of variance.

5 Explain the concept of confounding in factorial experiment.

6 Define main effects and interactions in 3^2 factorial experiments.

7 Define B.I.B.D. Also define two associate P.B.I.B.D's.

8 Define simple lattice design.

PART – B (4x15 = 60 Marks)
Answer any four questions.

9 What do you meant by selecting the best regression equation. Explain the following methods of selecting but regression equations:

i) All possible regression method using R^2 criterion.

ii) Stepwise regression procedure.

10 Explain Aitkens generalised least squares method.

11 Explain in detail principles of experimental designs.

12 Discuss the analysis of covariance of two way classification.

13 Give the method of construction of blocks in 3^3 partially confounding experiment for confounding NP^2K and NP^2K^2 parts of NPK interaction one in each replication.

14 Explain plain factorial experiments, also their advantages and drawback. Give the method of analysis of 2^4 factorial experiment by Yate's algorithm.

15 Explain B.I.B.D. model estimation, also give the intrablock analysis.

16 Prove the parametric relations in partially balanced block designs and derive the treatment sum of square (adjusted) in P.B.I.B.D with 2 associate classes.

17 Explain split plot design and give the method of analysis by deriving the sum of squares.

18 Explain analysis of complete RBD.

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Code No. 1056 / CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Mathematics

Paper – III: Operation Research and Numerical Techniques

Time: 2 Hours **Max.Marks:80**

PART – A (4x5 = 20 Marks)
Answer any four questions.

1 Write the characteristics of the standard four of LPP. Also define surplus variable in LPP.
2 State and prove reduction theorem of assignment problem
3 Define:
i) Saddle point
ii) Optimal strategies
iii) Value of the game.
4 Write the rules for drawing network diagram.
5 Explain secant method.
6 Explain the partition method.
7 Construct a difference table from the following values:

x	1	2	3	4	5	6	7	8	9
f(x)	1	8	27	64	125	216	343	512	729

8 Derive Simpson's 1/3 rule for integration.

PART – B (4x15 = 60 Marks)
Answer any four questions.

9 Solve the LPP by two-phase method:
Max $Z = 5x_1 + 8x_2$
STC $3x_1 + 2x_2 \geq 3$
$x_1 + 4x_2 \geq 4$
$x_1 + x_2 \leq 5$ and
$x_1, x_2 \geq 0$

10 Find the optimum solution to the following transportation problem.

To

		A	B	C	D	E	ai
From	I	3	4	6	8	8	20
	II	2	10	1	5	30	30
	III	7	11	20	40	15	15
	IV	2	1	9	14	18	13
b's	→	40	6	8	18	6	

11 Solve the game graphically whose payoff matrix for the player A is given below:

B

		I	II
A	I	2	4
	II	2	3
	III	3	2
	IV	-2	6

12 A project has the following characteristics.

Activity	1-2	2-3	2-4	3-5	4-5	4-6	5-7	6-7	7-8	7-9	8-10	9-10
to:	1	1	1	3	2	3	4	6	2	5	1	3
tp	5	3	5	5	4	7	6	8	6	8	3	7
tm	15	2	3	4	3	5	5	7	4	6	2	5

Construct a PERT network. Find critical path and variance.

13 Perform five iterations of the Muller's method to find the root of the equation.

$$f(x) = \cos x - xe^{x} = 0$$

14 Find the solution, to three decimals, of the system

83x + 11y – 4z = 95

7x + 52y + 13z = 104

3x + 8y + 29z = 71

Using Gauss-Seidel Method.

15 Derive Newton's Interpolation formulae. From the following table, find the number of students obtaining less than 45 marks.

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

16 Evaluate $\int_{4}^{52} \log x \, dx$ using

i) Trapezoidal Rule

ii) Simpson's Rule.

17 Solve the following Assignment problem.

Jobs

Machines	I	II	III	IV	V
A	11	10	18	5	9
B	14	13	12	19	6
C	5	3	4	2	4
D	15	18	17	9	12
E	10	11	19	6	14

18 Find the numerical solution of the simultaneous equations:

$$\frac{dx}{dt} + 2x + 3y = 0$$

$$3x + \frac{dy}{dt} + 2y = 2e^{2t}$$

with conditions at t=0, x=1, y=2 at the point t=0.1 by Tayler's method.

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Code No. 1048/O/BL

FACULTY OF SCIENCE

M. Sc. (Final) CDE Examination, November 2020

Subject : Mathematics

Paper – III : Elementary Number Theory

Time : 2 Hours **Max. Marks: 100**

Note: Answer any four questions. **(4x25=100)**

1 (a) If P_n is n^{th} prime, prove that the infinite series $\sum_{n=1}^{\infty} \frac{1}{p_n}$ diverges.

(b) State and prove division algorithm.

2 Prove that set of all multiplicative functions is a group under Dirichlet product of functions.

3 (a) State and prove Lagrange's theorem for polynomial congruences modulo P where P is a prime.

(b) State and prove Wolstenholme's theorem.

4 (a) State and prove principle of cross classification.

(b) Assume $m_1, m_2, \ldots m_r$ are relatively prime in pairs. Let $b_1, b_2, \ldots b_r$ be arbitrary integers and let $a_1, a_2, \ldots, a_r$ satisfy $(a_k, m_k) = 1$ for k = 1, 2, …. r . Then prove that the linear system of congruences

$$\begin{aligned} a_1 x &\equiv b_1 \pmod{m_1} \\ a_2 x &\equiv b_2 \pmod{m_2} \\ &\vdots \\ a_r x &\equiv b_r \pmod{m_r} \end{aligned}$$

has exactly one solution modulo $m_1 m_2 \ldots m_r$.

5 (a) State and prove Gauss Lemma.

(b) State and prove Quadratic reciprocity law.

6 (a) Let P be an odd prime and let d be any positive divisor of (P – 1). Then prove that in every reduced residue system mod P there are exactly $\phi(d)$ numbers "a" such that $\exp_p(a) = d$. In particular, when $d = \phi(P) = P - 1$ there are exactly $\phi(P - 1)$ primitive roots mod P.

(b) Let x be an odd integer. If $\alpha \geq 3$ prove that $x^{\phi(2\alpha)/2} \equiv 1 \pmod{2^{\alpha}}$ and hence show that there are no primitive roots mod 2^{α}.

7 (a) State and prove Reciprocity law for Jacobi symbols.

(b) If $n \geq 1$, prove that $P(n) < e^{k\sqrt{n}}$ where $K = \prod (2/3)^{1/2}$.

8 State and prove Euler's pentagonal number theorem.

9 For | x | < 1, prove that

$$\prod_{m=1}^{\infty} \frac{1}{1-x^m} = \sum_{n=0}^{\infty} P(n)x^n \quad \text{Where } P(0) = 1$$

10 State and prove Jacobi's triple product identity.

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Code No. 1071/CDE

FACULTY OF SCIENCE

M. Sc. (CDE) Examination, November 2020

Subject : Statistics

Paper – III : Operation Research

Time : 2 Hours **Max. Marks: 80**

Note: Answer any four questions. **(4x5=20 Marks)**

1 Define standard linear programming problem. Give the algorithm for its graphical solution.
2 Explain about models and modeling in OR.
3 What is an unbalanced transportation problem? Explain how to resolve it.
4 Explain Dominance property.
5 Write about factors affecting inventory control.
6 Define a game and explain operating characteristics of a queuing system.
7 What is goal programming problem? Explain, how it is different from linear programming problem.
8 What is sequencing problem? Give two examples.

PART – B

Note: Answer any four questions. **(4x15=60 Marks)**

9 Explain the concept of duality. Write Dual-Simplex algorithm.

10 Explain the rule of steepest ascent and ‘θ’ rule.

11 Explain forward pass and backward pass calculation procedures. Also define critical path.

12 Define Assignment problem. Give Hungarian algorithm to solve it.

13 Derive distribution of arrivals in a queuing theory.

14 Define different costs involved in inventory models. Derive the expressions for EoQ in an inventory model of single period without setup cost, instantaneous demand and units are continuous.

15 Define Integer programming problem. Give Gomery’s cutting plane algorithm to solve a pure integer programming problem.

16 What is Dynamic programming problem. Explain about characteristics of Dynamic programming

17 Distinguish PERT and CPM.

..2

18 For any 2 x 2 two-person zero-sum game without any Saddle point having the pay-off matrix for player

$$A \quad \begin{matrix} & B_1 & B_2 \\ A_1 & a_{11} & a_{12} \\ A_2 & a_{21} & a_{22} \end{matrix}$$

the optimum mixed strategies are $S_A = \begin{bmatrix} A_1 & A_2 \\ p_1 & p_2 \end{bmatrix}$ and $S_B = \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix}$ are determined by

$\frac{P_1}{P_2} = \frac{a_{22} - a_{21}}{a_{11} - a_{12}}$, $\frac{q_1}{q_2} = \frac{a_{22} - a_{12}}{a_{11} - a_{21}}$, where $P_1 + P_2 = q_1 + q_2$. Then show that value of the game to A is given by $v = \frac{a_{11}a_{22} - a_{21}a_{12}}{a_{11} + a_{22} - (a_{12} + a_{21})}$

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Code No. 1057 / CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Mathematics

Paper – IV: Fluid Mechanics

Time: 2 Hours **Max.Marks: 80**

PART – A (4x5 = 20 Marks)
Answer any four questions.

1 Write Newton's Laws of motion in detail.
2 State and prove conservation of angular momentum.
3 Discuss uniform flow post a stationary cylinder.
4 Define elliptic coordinates.
5 Write elementary properties of Vortex motion.
6 Derive the differential equation of steady viscous flow in tube of uniform cross-section.
7 What is boundary layer separation?
8 Describe the concept of prandtl boundary layer theory.

PART – B (4x15 = 60 Marks)
Answer any four questions.

9 Derive equation of continuity in cylindrical coordinates.

10 Derive Euler's equation of motion in the form

$$\frac{d\overline{q}}{dt} + (\overline{q}.\nabla)\overline{q} = -\frac{1}{p}\nabla\overline{p}\ .$$

11 Discuss steady flow between two co-axial cylinders.

12 Derive Navier-Stoke's equation of motion.

13 Discuss the motion due to circular and rectilinear vertices.

14 Discuss the motion of fluid in a uniform triangular cross-section under constant pressure gradient.

15 Explain: i) Dynamical similarity, ii) Boundary layer thickness, iii) Energy thickness.

16 Derive Van Karman's integral equation.

17 State and prove Blasius theorem.

18 Discuss the motion of flow between two parallel plates.

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Code No. 1062 /O /BL

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Mathematics

Paper – IV: Integral Transforms Integral Equations & Calculas of Variations

Time: 2 Hours **Max.Marks: 100**

Note: Answer any four questions. **(4x25=100)**

1 a) Prove that $L\,\dfrac{\sin t}{t}$ = $\tan^{-1}$ P.

b) Find $L^{-1}\left[\log\left(1-\dfrac{1}{p^2}\right)\right]$.

c) Solve $y'' + y' + 4t\,y = 0$ if y(0) = 3, $y'(0) = 0$.

2 a) Find Fourier sine transform of $f(x)$ = sin a x.

b) Find the Hankel transform of e^{-ax}/x, taking $xJ_0(px)$ as the Kernel of the transformation.

3 a) Find an integral equation corresponding to the differential equation $y'' + y$ = cos x, y(0) = $y'(0) = 0$.

b) Find the resolvent Kernel of the volterra integral equation with Kernel e^{x-t}.

4 a) Solve the integral equation $\varphi(x) = 2\int_0^1 xt\ \varphi^3(t)\ dt$.

b) Solved the integral differential equation $\varphi(x) = e^x + 2\int_0^x \cos(x-t)\ \varphi(t)\ dt$.

5 a) Solve the integral differential equation

$$\varphi''(x) + 2\phi(x) - 2\int_0^x \sin(x-t)\ \varphi'(t)\ dt = \cos x, \varphi(0) = \varphi'(0) = 0.$$

b) Solve the Volterra integral equation using the method of successive approximation.

6 Solve the boundary value problem using Green's function

$$y'' + y = x^2, y(0) = y(\pi/2) = 0.$$

7 Show that all iterated Kernels of a symmetric Kernels are symmetric.

8 Find the extremal of the functional $V(y(x)) = \int_0^1 (1 + y''^2)dx$ subject to conditions

y $y(0) = 0,\ y'(0) = y(1) = y'(1) = 1.$

9 Define Brahistochrone problem and obtain the solution of the same in the form of a cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$

10 Derive the equation of motion of simple pendulum using Lagrange's equation.

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Code No. 1072/CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Statistics

Paper – IV : Time Series Analysis, Statistics Process and Quality Control

Time: 2 Hours **Max.Marks: 80**

PART – A (4x5 = 20 Marks)
Answer any four questions.

1 Explain weighted moving average smoothing method.
2 Define two equivalent forms of linear stationary process.
3 Define ARIMA (p, d, q) and ARIMA (1, 1, 1) processes.
4 What are initial estimates? What is the use of it?
5 Explain about average run length curve. Give its formula.
6 State advantages of control charts for attributes over control charts for variables.
7 Describe Rectifying Sampling Procedures. Give an example.
8 Describe multiple sampling plans.

PART – B (4x15 = 60 Marks)
Answer any four questions.

9 Describe in detail about periodogram analysis of time series.

10 Define Moving Average Process of order ‘q’ and derive its properties.

11 Derive general expressions for ψ – weights and π – weights.

12 Derive parameter estimates of Auto Regressive model by the method of maximum likelihood estimation.

13 Give a detailed explanation on economic designing of X-chart.

14 Derive the control limits for moving average chart for mean and range.

15 Derive expression for ‘n’ in sampling plans for variables when upper specification limit is given.

16 Explain how to design a sequential sampling plan can be designed.

17 Explain Holt and Holt-winter exponential smoothing methods.

18 Define exponentially weighted moving average. Derive control limits based on the same statistic.

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Code No. 1058 / CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Mathematics

Paper – V: Integral Transforms Integral Equations & Calculus of Variations

Time: 2 Hours **Max.Marks: 80**

PART – A (4x5 = 20 Marks)

Answer any four questions.

1 Prove that $L\left[\frac{\sin t}{t}\right] = \tan^{-1}(p).$

2 Find Fourier sin e transform of e^{-x}.

3 Find the resolvent Kernel for Volterra integral equation with Kernel $k(x,t) = \frac{2+\cos x}{2+\cos t}.$

4 Solve the integral equation by using the method of successive approximation. $\varphi(x) = x + 1 - \int_o^x \varphi(t)\,dt, \varphi_0(x) = 1.$

5 Solve the integral equation $\varphi(x) = 2\int_0^1 xt \quad \varphi^3(t)dt.$

6 Solve the boundary value problem using Green's function $y'' + y = 0,\ y(0) = y\left(\frac{\pi}{2}\right) = 0.$

7 State and prove the fundamental lemma of calculus of variation.

8 What is the problem of the Brachisto chrone?

PART – B (4x15 = 60 Marks)

Answer any four questions.

9 i) Find $L^{-1}\left[\frac{p-1}{(p+3)(\mathrm{p}^2+2p+2)}\right].$

ii) Solve $[tD^2 + (1-2t)D - 2] > 0 \ if \ \ y(0) = 1, y'(0) = 2.$

10 i) Find Fourier cosine transform of f(x) = sin x

ii) Find Hankel transform of e^{-ax} taking $xJ_0(px)$ as the Kernel of the transformation.

11 Transform the problem into integral equation $\frac{d^2y}{dx^2} + \lambda y = f(x),\ y(0) = 1,\ \ y'(0) = 0$

12 Solve the integro-differential equation

$$\varphi''(x)+\int_0^x e^{2(x-t)}\varphi'(t)\,dt = e^{2x},\ \varphi(0)=0,\ \varphi'(0)=1.$$

13 Find the characteristic numbers and eigen functions of the integral equation.

$$\varphi(x)=\lambda\int_0^{\pi}\left(\cos^2 x\ \cos 2t+\cos 3x\ \cos^3 t\right)\varphi(t)\,dt = 0\ .$$

14 Construct the Green's function for the homogeneous B.V.P.

$$y^{iv}(x)=0,\ y(0)=y'(0)=y(1)=y'(1)=0.$$

15 Explain the isoperimetric problem and find a variational solution to it.

16 Derive the Euler-Poisson equation.

17 Derive Hamilton's canonical equation of motion.

18 Solve the equation $\varphi(x)-\lambda\int_0^{\pi}\left(\cos^2 x\cos 2\ t\right)+\cos^3 t\ \cos 3\ x)\ \varphi(t)\,dt$ =0.

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Code No. 1073 / CDE

FACULTY OF SCIENCE

M.Sc. (Final Practical) CDE Examination, November 2020

Sub: Statistics

Practical Paper – I: Statistical Inference Linear Models & Design of Experiments

Time: 2 Hours **Max.Marks: 100**

Note: Answer any two questions. All questions carry equal marks. Scientific calculation and allowed.

1 a) An experimental engine operated for 24, 28, 21, 23, 32 and 22 minutes with a gallon of a certain kind of fuel. Assuming normality, test the hypothesis $H_0: \mu = 29$ Vs $H_1: \mu < 29$, where μ is the mean operative time. Take $\alpha = 0.01$.

b) On the assumption that the distribution is normal with variance unity, examine by a sequential ratio test procedure whether the mean is zero against the alternative that it is 0.5 for the data: 2.12, 1.32, 0.81, -1.91, -1.23, -0.17, -0.38, -0.03, -1.65, -0.35, -0.99, -0.67, -0.28, 0.40, -0.12, -0.10, -0.83, -0.37, -0.48. Take $\alpha = 0.05$ and $\beta = 0.15$.

2 a) The following are the average weekly losses of worker hours due to accidents in 10 industrial plants before and after a certain safety program was put into operation:

Before	45	73	46	124	33	57	83	34	26	17
After	36	60	44	119	35	51	77	29	24	11

Use the 0.05 level of significance to test whether the safety program is effective.

b) In an industrial production line, items are inspected periodically for defectives. The following is a sequence of defective items, D, and non-defective items, N produced by this production line.

DDNNN DNN DDNNNNN DDDNN DNNNN DND.

Carry out such test at $\alpha = 0.05$, to determine whether the defectives are occurring at random.

3 Carry out Friedman test for the following data on mileage of cars for type of gasoline and brand of cars.

Car Brand

		A	B	C
Tyre of gasoline	I	22.4	17.0	19.2
	II	20.8	19.4	20.2
	III	21.5	18.7	21.2

Test at α = 0.05.

4 The following data pertains to nitrous oxide (y), humidity (x_1) and temperature (x_2) of air in a location

y	0.9	0.91	0.96	0.89	1.15	0.77
x_1	72.4	41.6	34.3	35.1	8.3	72.2
x_2	76.3	70.3	77.1	68.0	66.8	77.7

i) Fit the model y = $\beta_o + \beta_2 x_2$ + ε using least squares method of estimation.

ii) Test a) H_o: $(\beta_1 \;\; \beta_2)' = (0 \;\; 0)'$ and b) H_o: β_2 = 0.

5 The following data are obtained on time of reaction for a chemical process. The experiment was conducted using a BIBD.

Batch of raw materials (blocks)

		B1	B2	B3	B4
Catalyst (Treatment)	1	73	74	-	71
	2	-	75	67	72
	3	73	75	68	
	4	75	-	72	75

Carryout the analysis and draw your conclusions. Find the standard error of the BLUE of any two treatment.

6 The following is the data on hours of operation of stepping switch. The experiment was conducted as a 2^3 factorial experiment confounding one effect in each replicate.

Replicate-I				Replicate-II			
Block 1		Block 2		Block 3		Block 4	
(1)	50	a	75	abc	68	(1)	48
ab	62	B	63	a	78	ab	56
c	71	ac	523	b	55	ac	45
abc	55	bc	48	c	62	bc	35

Identify the confounded effects and carry out the analysis.

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